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M.A./M.Sc. (Third Semester)
EXAMINATION, Dec. - Jan., 2021-22
MATHEMATICS
Paper Second
(Partial Differential Equations and Mechanics -I)

Time : Three Hours] ***[Maximum Marks : 80***

Note : Attempt all sections as directed.

Section - A

(Objective/Multiple Choice Questions)

(2 Marks each)

Note: Attempt all questions.

Choose the correct answers:

1. The function : $\phi(x,t) = \begin{cases} \frac{1}{(4\pi t)^{n/2}} e^{\frac{-|x|^2}{4t}} & x \in R^n,\ t > 0 \\ 0 & x \in R^n,\ t < 0 \end{cases}$

is fundamental solution of.

(A) Laplace equation

(B) Wave equation

(C) Transport equation

(D) Heat equation

2. If $U(x,y)$ is a harmonic function in $R^2, P_0(x_0,y_0) \in R^2$, $K_a = \{P \in R^2 : |P - P_0| \le a\}$ and $C_a = \partial K_a$. Then U (P_0) =

(A) $\frac{1}{4\pi a^2} \iint_s U(P)dS_p$

(B) $\frac{1}{4\pi a} \oint_{c_a} U(P)dS_p$

(C) $\frac{1}{2\pi a} \oint_{Ca} U(P)dS_p$

(D) $\frac{1}{2\pi a} \oint_{Ka} U(P)dS_p$

3. If U(P) be a harmonic function in the domain Ω and U be bounded from above. Then which of the following is true?

(A) If U attains sup U in Ω then U is constant.

(B) If U is constant then U attains sup U in Ω.

(C) Both (A) and (B)

(D) None

4. The PDE $U_t + b\,DU = 0$ represents -

(A) Laplace's equation

(B) Wave equation

(C) Heat equation

(D) Transport equation

5. The constrains involved in the motion of rigid bodies, are

   (A) Holonomic

   (B) Non - Holonomic

   (C) Both (A) and (B)

   (D) None

6. For conservative systems, the Hamiltonian function H =

   (A) K.E.

   (B) P.E.

   (C) K.E. + P.E.

   (D) None

7. If linear momentum $\overrightarrow{P} = P_x\hat{i} + P_y\hat{j} + P_Z\hat{k}$ and angular momentum $\overrightarrow{L} = L_x\hat{i} + L_y\hat{j} + L_Z\hat{k}$ then poisson bracket $[P_Z, L_y] =$

   (A) P_Z

   (B) P_y

   (C) $-P_x$

   (D) P_x

8. The path of a particle, sliding from one point to another in the absence of friction in the shortest time is:

   (A) Straight line

   (B) Circle

   (C) Cycloid

   (D) Catenary

9. Attraction of a disc of infinite radius and small thickness k at a point on the axis of the disc at a distance P is -

   (A) $2\pi\gamma k\varrho\left[1-\frac{P}{\sqrt{a^2+p^2}}\right]$

   (B) $4\pi\gamma k\varrho$

   (C) $2\pi\gamma k\varrho$

   (D) $3\pi\gamma k\varrho$

10. If V is the potential of an attracting system at any point $P(x, y, z)$ which does not coincide with any of the attracting particles then: $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial Z^2} =$

    (A) $4\pi\gamma\varrho$

    (B) $-4\pi\gamma\varrho$

    (C) $-2\pi\gamma\varrho$

    (D) 0

Section - B

(Very Short Answer Type Questions)

(2 Marks each)

Note : Attempt all questions.

1. Write the statement of Poisson's theorem in polar coordinates:

2. Define generalised momentum.

3. Write Hamilton's principle.

4. Define Poisson Brackets.
5. Write integral of motion in Poisson Bracket form.
6. Write maximum principle for Laplace's equation.
7. Wrie Harnack's inequality.
8. Write relation between attraction and potential.

Section - C

(Short Answer Type Questions)

(3 Marks each)

Note : Attempt all questions.

1. Find the attraction of a thin uniform spherical shell at an external point.
2. Determine the potential of a thin uniform circular disc.
3. State and prove conservation theorem for generalised momentum.
4. Show that the Hamiltonian function H represents the total energy of the system.
5. State and prove Jacobi identity for poisson brackets.
6. If $\phi(x,t)$ is the fundamental solution of the Heat equation then show that $\int_{R^n} \phi(x,t)dx = 1$
7. Two particles of masses m_1 and m_2 more under the action of their gravitational interaction. Find the Lagrangian equation.
8. Prove that the shortest distance between two points in a plane is a straight line.

Section - D

(Long Answer Type Question)

(5 Marks each)

Note : Attempt all questions.

1. Derive D'Alembert's formula.

OR

State and prove mean value formula's for Laplace's equation.

2. Derive fundamental solution of Laplace's equation.

OR

If $U \in C(U)$ satisfies the mean value property for each ball $B(x,r) \subset U$ then show that $U \in C^{\infty}(U)$.

3. Show that Poisson Bracket under canonical transformation is invariant.

OR

State and prove Hamilton's canonical equations.

4. Prove that the attraction of a thin uniform cylindrical shell of radius a and length ℓ at a point on its axis at a distance b from one end, $\ell - b$ from the other $\left(b < \frac{\ell}{2}\right)$ is:

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$$y\frac{m}{2\pi al}\left[\frac{1}{\sqrt{a^2+b^2}}-\frac{1}{\sqrt{a^2+(l-b)^2}}\right]$$

where M is the mass of the shell, the ends of the shell are open and circular.

OR

Show that the attraction of a solid hemisphere at the centre of its plane base is $\frac{3}{2}\frac{\gamma M}{a^2}$, where M is the mass and a is the radius.